

The Art of Problem Solving
Post-Test
Introduction to Counting & Probability

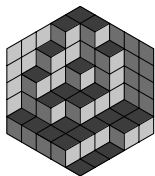
If you can solve all of the following problems *with little difficulty*, then the book **Introduction to Counting & Probability** would largely serve as a review for you.

Answers to these problems are on the following page. **Do not use a calculator.**

1. How many multiples of 7 are between 83 and 229?
2. How many distinct arrangements are there of the letters in the word MATHEMATICS?
3. A coin is flipped, a 6-sided die numbered 1 through 6 is rolled, and a 10-sided die numbered 0 through 9 is rolled. What is the probability that the coin comes up heads and the sum of the numbers that show on the dice is 8?
4. Find the coefficient of x^3y^8 in the expansion of $(x - 2y^2)^7$.
5. Particle Man is at the origin in three-dimensional space. How many ways can Particle Man take a series of 12 unit-length steps, each step parallel to one of the coordinate axes, from the origin to $(3, 4, 5)$ without passing through the point $(2, 3, 2)$?
6. In poker, a hand is formed with 5 cards. The deck has 52 cards, separated into 4 suits. Each suit has 13 ranks which are the same in every suit. A full house occurs when a hand has 3 cards of one rank and 2 of another. How many different poker hands are full houses?
7. How many distinguishable ways can the faces of a regular hexagonal prism be painted 8 different colors (one color per face, no color used twice)?
8. There are $2n$ players in a chess tournament. The first round consists of pairing the players to participate in n matches with every player playing one match. In terms of n , how many ways can this pairing take place?
9. Find two proofs that for every positive integer n , the following equality holds:

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \dots + (-1)^n \binom{n}{n} = 0.$$

10. A playoff series between two teams proceeds one game at a time until one team has won 5 games. What is the probability that the series lasts 9 games if each team is equally likely to win each game?



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Answers

1. 21
2. 4989600
3. $1/20$
4. 560
5. 23520
6. 3744
7. 3360
8. $\frac{(2n)!}{2^{n}n!}$
9. One method is to let $x = y = 1$ in the binomial expansion of $(x - y)^n$. There are many others.
10. $35/128$