



## Index

- $\Leftrightarrow$ , 231
- $\sphericalangle$ , 13
- $\pi$ , 285
- 3-D Geometry, 356–407
- 30-60-90 triangle, 144
- 45-45-90 triangle, 142
  
- AA Similarity, 101
  - proof of, 119
- AAS Congruence, 61
- acute, 18
- adjacent angles, 19
- alternate exterior angles, 27
- alternate interior angles, 27
- altitudes, foot of, 86
- AMC, vii
- American Mathematics Competitions, *see* AMC
- American Regions Math League, *see* ARML
- analytic geometry, 435–477
- angle bisector, 171
- Angle Bisector Theorem, 181, 201
- angle of depression, 502
- angle of elevation, 502
- Angle-Angle Similarity, 101
  - proof of, 119
- Angle-Angle-Side Congruence Theorem, 61
- angle-chasing, 29
- Angle-Side-Angle Congruence Theorem, 61
- angles, 13–48
  - acute, 18
  - adjacent, 19
  - alternate exterior, 27
  - alternate interior, 27
  - between a tangent and a chord, 319
  - between a tangent and a secant, 320
  - between two chords, 312
  - between two secants, 313
  - central, 286
  - complementary, 22
  - corresponding, 27
  - dihedral, 359
  - exterior, 36
  - inscribed, 304–310, 320
  - interior, 36
  - reflex, 19
  - remote interior, 37
  - right, 17
  - same-side exterior, 27
  - same-side interior, 27

- straight, 21
- supplementary, 22
- trisection, 338
- vertical, 23
- Annairizi of Arabia, 162
- apex, 366
- apothem, 252
- arc, 6, 284
- Archimedes, 289, 293, 407
- area, 83–88
  - similar figures, 117
- ARML, viii
- Art of Problem Solving, v, 562
- ASA Congruence, 61
- axiom, 10
  
- base, of a triangle, 86
- bisects, 54
- buckminsterfullerene, 401
  
- Cartesian plane, 436
- central angle, 307
- centroid, 183
- chord, 6
  - bisected by radius, 54
- circles, 5, 284–304
  - area, 290
  - center, 5
  - chord, 6, 340
  - circumference, 284
  - diameter, 6
  - radius, 5
  - secant, 340
  - tangent, 340
- circular segment, 294, 295
- circumcenter, 175
- circumcircle, 175
- circumference, 284
- circumradius, 175
- circumscribed quadrilateral, 327
- collinear, 4
- compass, 7
- complementary angles, 22
- concave, 206
- concurrent, 4, 174
- cones, 385–390
  
- congruent, 49
- conjecture, 10
- construction, 7
  - angle bisector, 197
  - copy an angle, 121
  - equilateral triangle, 72
  - parallel lines, 121
  - perpendicular bisector, 73
  - perpendicular lines, 161
- contradiction, 41, 143
- converse, 40
- convex, 206
- coordinate geometry, 435–477
- coordinates, 436
- coplanar, 358
- corresponding angles, 27
- cosecant, 487
- cosine, 480
- cot, 487
- cotangent, 487
- CPCTC, 54
- cross-section, 364
- csc, 487
- cube, 363
- cyclic quadrilateral, 322, 323
- cylinders, 380–384
  - axis, 380
  
- degenerate triangle, 275
- degree, 14
- Dehn, Max Wilhelm, 369
- Descartes, René, 435
- diagonal, 245
- diagonals, 207
- diameter, 6
- dihedral angle, 359
- dilation, 418–422
- dimensions, 2
- distance formula, 441, 473
- dodecahedron, 372
- doubling the cube, 338
  
- eight-point circle, 212
- Elements*, 189
- endpoints, 2
- equidistant, 3

## INDEX

- 
- equilateral triangle, 68
  - Eratosthenes, 302
  - Euclid, 189
  - Euler line, 191
  - Euler, Leonhard, 191
  - excenter, 205
  - excircle, 205
  - extradius, 205
  - Extended Law of Sines, 500
  - exterior angles, 36, 246
  
  - face diagonal, 362
  - Fagnano's Problem, 423
  - Fermat numbers, 339
  - Fermat prime, 339
  - Fermat's Last Theorem, 151
  - Fermat, Pierre de, 151
  - Fibonacci sequence, 19
  - fixed point, 409
  - foot, 86
  - frustum, 379
    - right circular, 389
  
  - Garfield, President James A. , 316
  - generalization, 496
  - geometric mean, 117
  - golden ratio, 12, 19
  - Golden Ratio Spiral, 12
  - golden rectangle, 12
  - graph, 437
  
  - Harvard-MIT Math Tournament, *see* HMMT
  - height, 86
  - hemisphere, 407
  - Heron's Formula, 159
  - Hilbert, David, 56, 369
  - Hippasus, 143
  - HL Congruence, 153
  - HL Similarity, 155
  - HMMT, vii
  - homothety, 422
  - hypotenuse, 133
  - Hypotenuse-Leg Congruence, 153
  - Hypotenuse-Leg Similarity, 155
  
  - icosahedron, 372
  - identity, 409
  
  - if and only if, 227
  - iff, 231
  - image, 408
  - incenter, 180
  - incircle, 180
  - Inner Napoleon Triangle, 214
  - inradius, 180
  - inscribed angles, 304–320
  - interior angles, 245
  - invariants, 369
  - inversion, 430
  - irrational numbers, 143, 285
  - isosceles trapezoid, 213
  - isosceles triangle, 67
  
  - kite, 240
  
  - lateral surface area, 361
  - lattice points, 436
  - Law of Cosines, 494–500
  - Law of Sines, 494–500
  - Leg-Leg Congruence, 153
  - Leg-Leg Similarity Theorem, 155
  - legs, of a right triangle, 133
  - Lincoln, Abraham, 189
  - line, 3
  - line of symmetry, 414
  - line segment, 2
  - linear equation, 437
  - LL Congruence, 153
  - LL Similarity, 155
  - locus, 5
  - lune, 294, 296
  
  - major arc, 6
  - Mandelbrot Competition, vii
  - map, 408
  - MATHCOUNTS, vii
  - medial triangle, 185
  - medians, 183
  - Midline Theorem, 186
  - midpoint, 3
  - Millennium Problems, 216
  - minor arc, 6
  - Mohr-Mascheroni Theorem, 339
  
  - Napoleon's Triangles, 214

- nine-point circle, 193
- noncollinear, 176
- noncoplanar, 358
- nondegenerate triangles, 275
  
- obtuse, 18
- octahedron, 372
- octave, 157
- ordered pair, 436
- origin, 3, 436
- orthocenter, 189
- orthodiagonal, 236
- Outer Napoleon Triangle, 214
  
- parallel lines, 24
- Parallel Postulate, 26
- parallelepiped, 360
- parallelograms, 215–219
- parity, 132
- Penrose tilings, 265
- pentomino, 131
- Perelman, Grigori, 216
- perimeter, 81–83
- perpendicular, 17
- perpendicular bisector, 72, 73, 170
- perpendicular, foot of, 86
- pi, 285
- plane of symmetry, 418
- planes, 4, 356–359
  - parallel, 358
  - perpendicular, 359
- Plato, 373
- Platonic solids, 373
- Poincaré Conjecture, 216
- point, 2
- point at infinity, 430
- point-slope form, 439, 473
- polygons, 244–266
  - angles, 246–249
  - area of a regular, 252
  - interior angles, 245
  - regular, 245
- polyhedra, 360
  - regular, 370–374
- polyhedron, *see* polyhedra
- polyomino, 131
  
- Poncelet-Steiner Theorem, 339
- postulates, 10
- Power of a Point, 340–355
- prisms, 360–366
- problem solving, iii
- proof by contradiction, 41, 143
- protractor, 14
- pyramids, 366–369
- Pythagoras, 142
- Pythagorean Theorem, 134–140, 147–152, 163
- Pythagorean triple, 147
- Pythagoreans, 142
  
- QED, 311
- quadrants, 490
- quadrilaterals, 206–243
  - circumscribed, 327
  - concave, 206
  - cyclic, 323
  - diagonal, 207
  - parallelogram, 215
  - rectangle, 221
  
- radian, 289
- radical axis, 352
- Radical Axis Theorem, 352
- radius, 5
  - bisects a chord, 54
- Ramanujan, 298
- rational numbers, 143
- ray, 3
- rectangles, 83, 221–224
  - area, 85
- reflections, 310, 413–418
- reflex angles, 19
- remote interior angles, 37
- resources, v
- rhombi, 219–221
- rhombus, *see* rhombi
- right angle, 17
- right rectangular prism, 360
- right triangles, 84, 133–169
  - area, 84, 85
  - inradius of, 327
- rotations, 411–413
  
- same-side exterior angles, 27

## INDEX

- 
- same-side interior angles, 27
  - SAS Congruence, 57
  - SAS Similarity, 111
  - scale factor, 418
  - scalene triangle, 71
  - sec, 487
  - secant, 310, 487
  - secant line, 6
  - sectors, 290, 291
  - segment, 2
  - semicircle, 14
  - semiperimeter, 159
  - Side-Angle-Side Congruence Theorem, 57
  - Side-Angle-Side Similarity, 111
  - Side-Side-Side Congruence Theorem, 53
  - Side-Side-Side Similarity, 113
  - similarity, 100
  - Simson line, 318
  - sine, 479
  - skew lines, 358
  - slant height, 366
  - slope, 437
  - slope-intercept form, 439, 473
  - SOHCAHTOA, 482
  - space, 4
  - space diagonal, 362
  - spheres, 390–394
    - surface area, 390
    - volume, 390
  - spherical geometry, 80
  - squares, 83, 224–226
  - squaring the circle, 338
  - SSA Congr. . .oops, 65
  - SSS Congruence, 53
  - SSS Similarity, 113
  - standard form, 439, 473
  - straight angle, 21
  - straightedge, 7
  - subtend, 287
  - supplementary angles, 22
  - symmetry, 52, 256
  - tangent, 6, 316, 480
    - common external, 325
    - externally, 278
    - internally, 279
    - length of common, 325
  - tangrams, 243
  - tessellation, 264
  - Thales of Miletus, 306
  - Thales Theorem, 306
  - theorems, 10
  - three-dimensional geometry, 356–407
  - tiling, 264
  - total surface area, 361
  - transformations, 408–434
  - translations, 408–410
  - transversal, 26
  - trapezoids, 208–214
    - base angles, 213
    - bases, 208
    - isosceles, 213
    - legs, 208
    - median, 208
  - Triangle Inequality, 274–279
  - triangles, 31
    - 30-60-90, 144
    - 45-45-90, 142
    - altitudes, 86, 188–192
    - angle bisectors, 177–182
      - area, 87
      - centroid, 183
      - cevians, 170–205
      - circumcircle, 175
      - congruent, 49–80, 152–156
      - degenerate, 275
      - equilateral, 68
      - height, 86
      - incenter, 180
      - incircle, 180
      - inradius, 180
      - isosceles, 67
      - medians, 183–187
      - nondegenerate, 275
      - orthocenter, 189
      - perpendicular bisectors, 173–177
      - right, 84, 85, 133–169
      - scalene, 71
      - similar, 99–133, 152–156
  - trigonometric identity, 484
  - trigonometry, 478–504

unit circle, 489

USA Mathematical Talent Search, *see* USAMTS

USAMTS, viii

vertex angle, 67

vertical angles, 23

volume, 360

Without loss of generality, 276

WLOG, 276

$x$ -axis, 436

$x$ -coordinate, 436

$x$ -intercept, 437

$y$ -axis, 436

$y$ -coordinate, 436

$y$ -intercept, 437