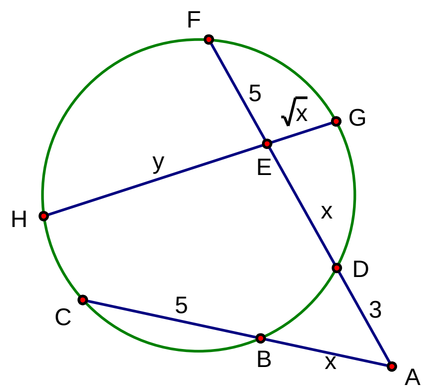
If you can solve all of the following problems *with little difficulty*, then the book **Introduction to Geometry** would largely serve as a review for you.

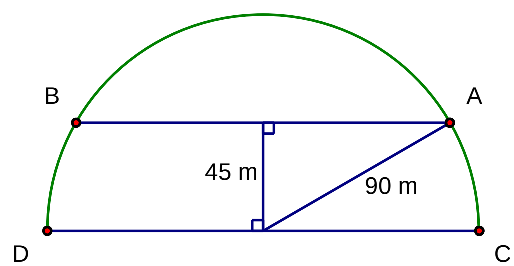
Answers to these problems are start on the third page. **Do not use a calculator.**

1. Prove the Pythagorean Theorem.

2. Find y in the diagram below.



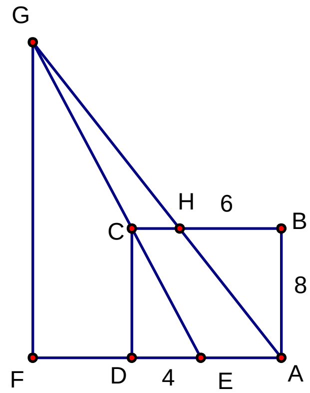
3. Marcia could walk from A to B along arc AB on the semicircular path, or she can walk along chord AB. Diameter CD has length $180m$. How much farther is it to walk along the arc as opposed to the chord?



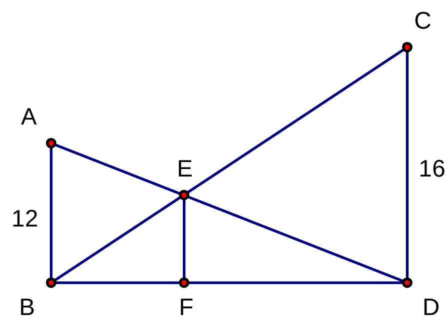
4. An ant starts at one vertex of a unit cube and walks to the opposite vertex along the surface of the cube. What is the minimum distance the ant can walk?

5. Spot's doghouse has a regular hexagonal base that measures one yard on each side. He is tethered to a vertex with a two-yard rope. What is the area, in square yards, of the region outside the doghouse that Spot can reach?

6. In rectangle $ABCD$, we have $AB = 8$, $BC = 9$, H is on BC with $BH = 6$, E is on AD with $DE = 4$, line EC intersects line AH at G, and F is on line AD with $GF \perp AF$. Find the length GF.



7. There are two flagpoles, one of height 12 and one of height 16. A rope is connected from the top of each flagpole to the bottom of the other. The ropes intersect at a point x units above the ground. Find x. In the accompanying diagram, this is equivalent to finding the length of EF.



8. Three spheres are tangent to a plane at the vertices of a triangle and are tangent to each other. Find the radii of these spheres if the sides of the triangle are 6, 8, and 10.

9. Derive a general formula for the volume of the frustum of a cone with bases of radius R and r and height h.

10. Describe how to construct a line tangent to two given circles using a straightedge and compass.

Answers

1. (Note that there are many acceptable proofs.) In right triangle ABC with right angle at A we wish to prove $AC^2 + AB^2 = BC^2$. Drop altitude AD to hypotenuse BC. $\triangle ABC \sim \triangle DAC \sim \triangle DBA$ giving us $\dfrac{DC}{AC} = \dfrac{AC}{BC}$ and $\dfrac{DB}{AB} = \dfrac{AB}{BC}$. Now $AC^2 = BC \cdot DC$ and $AB^2 = BC \cdot DB$, so $AC^2 + AB^2 = BC(DC + DB) = BC^2$.

2. 10

3. $60\pi - 90\sqrt{3}$

4. $\sqrt{5}$

5. 3π

6. 20

7. $\dfrac{48}{7}$

8. $r_1 = \dfrac{12}{5}, r_2 = \dfrac{15}{4}, r_3 = \dfrac{20}{3}$

9. $V = \dfrac{1}{3}\pi h(R^2 + Rr + r^2)$

Solution to #10 is on the next page

10.

Draw two chords in each circle construct the PERPENDICULAR BISECTORS and identify the center of each circle. These are points P and C in the diagram.
Draw radius PC in the smaller circle and construct a PARALLEL RADIUS QD in the larger circle.
Draw segment CD connecting the endpoints of the two radii, and construct parallel line PE through the center of the smaller circle.
We now have segment QE of length $R - r$, where R and r are the radii of the circles. Using this length as a radius, construct a circle concentric with the large circle.
Construct PF, a TANGENT TO THIS CIRCLE FROM THE CENTER OF THE SMALL CIRCLE.
Construct QG, a RADIUS OF THE LARGE CIRCLE PERPENDICULAR TO THIS TANGENT.
Construct radius PH of the small circle parallel to this newest radius, and connect the intersection points of these radii with their respective circles.
(Note that each of the items in all caps represents a separate construction you need to know how to do in order to do the overall construction.)