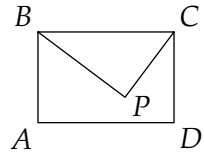


583. If (x, y) is a solution for the system of equations below, find the maximum value of $x^2 + y^2$. (MAӨ 1991)

$$2x^2 + 5xy + 3y^2 = 2$$

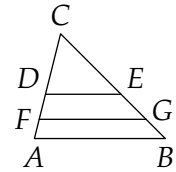
$$6x^2 + 8xy + 4y^2 = 3$$

584. In rectangle $ABCD$, $AB = 10$ and $BC = 15$. A point P inside the rectangle is such that $PB = 12$ and $PC = 9$. What is the length of PA ? (ARML 1981)



585. We are given 4 bags of coins such that (a) all coins in a given bag weigh the same, and (b) the coins of a given bag weigh either 1, 2, or 3 ounces. Take 1 coin from bag 1, 3 coins from bag 2, 9 coins from bag 3, and 27 coins from bag 4. Weighing these 40 coins together on a scale yields a weight of 95 ounces. Determine the weight of a coin from each of the 4 bags. (ARML 1982)

586. The two lines DE and FG are both parallel to AB , and the 3 regions CDE , $DEGF$, and $FGBA$ have equal areas. Find CD/FA . (ARML 1982)



587. How many distinct solutions (a, b, c) are there to the equation $a^2 + bc = b^2 + ac$ if a , b , and c are integers between 1 and 5 inclusive? (Mandelbrot #2)

588. $ABCD$ is a square with side of unit length. Points E and F are taken respectively on sides AB and AD so that $AE = AF$ and the quadrilateral $CDFE$ has maximum area. What is this maximum area? (AHSME 1962)

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