

*The Art of Problem Solving Online Classes*  
Are You Ready for  
**Intermediate Counting & Probability?**

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Students enrolling in **Intermediate Counting & Probability** should be comfortable with geometric series, factoring and multiplying polynomials, and basic counting techniques. Examples of each are below.

**Geometric Series.**

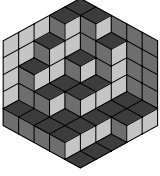
1. Evaluate the sum:  $1 + 2 + 2^2 + 2^3 + \dots + 2^{10}$ .
2. Assuming that  $-1 < x < 1$ , find a closed form expression for  $1 - x + x^2 - x^3 + x^4 - x^5 + \dots$  by evaluating the sum as an infinite geometric series with common ratio  $-x$ .

**Factoring and Multiplying Polynomials.**

3. Find the polynomial  $f(x)$  such that  $(x - 1)f(x) = x^6 - 1$ .
4. Find the five terms with smallest degree of the product  $(1 + x + x^2 + x^3 + x^4 + \dots)(1 + 2x + 3x^2 + 4x^3 + 5x^4 + \dots)$ .

**Counting Techniques.** The following questions are from the “Do You Know Introduction to Counting & Probability” quiz. If you cannot easily solve most of them, you should consider taking our **Introduction to Counting & Probability** course before taking **Intermediate Counting & Probability**.

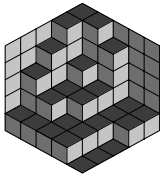
5. How many multiples of 7 are between 83 and 229?
6. How many distinct arrangements are there of the letters in the word MATHEMATICS?
7. A coin is flipped, a 6-sided die numbered 1 through 6 is rolled, and a 10-sided die numbered 0 through 9 is rolled. What is the probability that the coin comes up heads and the sum of the numbers that show on the dice is 8?
8. Find the coefficient of  $x^3y^8$  in the expansion of  $(x - 2y^2)^7$ .
9. Particle Man is at the origin in three-dimensional space. How many ways can Particle Man take a series of 12 unit-length steps, each step parallel to one of the coordinate axes, from the origin to  $(3, 4, 5)$  without passing through the point  $(2, 3, 2)$ ?
10. In poker, a hand is formed with 5 cards. The deck has 52 cards, separated into 4 suits. Each suit has 13 ranks which are the same in every suit. A full house occurs when a hand has 3 cards of one rank and 2 of another. How many different poker hands are full houses?
11. How many distinguishable ways can the faces of a regular hexagonal prism be painted 8 different colors (one color per face, no color used twice)?



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12. There are  $2n$  players in a chess tournament. The first round consists of pairing the players to participate in  $n$  matches with every player playing one match. In terms of  $n$ , how many ways can this pairing take place?
13. A playoff series between two teams proceeds one game at a time until one team has won 5 games. What is the probability that the series lasts 9 games if each team is equally likely to win each game?



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The answers to Are You Ready for Intermediate Counting & Probability are below. (The answers to problem sets and challenges given in the class will include full detailed solutions as opposed to the mere answers provided below.)

1. 2047
2.  $\frac{1}{1+x}$
3.  $x^5 + x^4 + x^3 + x^2 + x + 1$
4.  $1 + 3x + 6x^2 + 10x^3 + 15x^4 + \dots$
5. 21
6. 4989600
7.  $1/20$
8. 560
9. 23520
10. 3744
11. 3360
12.  $\frac{(2n)!}{2^n n!}$
13.  $35/128$